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Discussion Papers in
Economics and Econometrics

Nominal GDP targeting and the tax burden

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No. 1604

This paper is available on our website
http://www.southampton.ac.uk/socsci/economics/research/papers

ISSN 0966-4246

Nominal GDP targeting and the tax burden

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First draft: 14 Dec 2015. This version**:** 4 July, 2016.

Abstract

An overlapping generations model is set out in which monetary policy matters for distortionary taxes because unanticipated inflation has real wealth effects on households with nominal government debt. The model is used to study the tax burden under inflation and nominal GDP targeting. Nominal GDP targeting makes taxes less volatile than inflation targeting but raises average taxes. With a quadratic loss function, the expected tax burden is minimized with only indexed debt under inflation targeting, but with both indexed and nominal debt under nominal GDP targeting. Nominal GDP targeting lowers the tax burden relative to inflation targeting (except at very high indexation shares), but this conclusion hinges on risk aversion, productivity persistence and the loss function for the tax burden.

Keywords: nominal GDP targeting, inflation targeting, government debt, tax burden.
JEL classification: E52.

1 Introduction

Over the past few decades, monetary policy analysis in academia and central banks has focused mainly on New Keynesian models (see Woodford, 2003). In these models, monetary policy has the potential to raise social welfare by stabilizing inefficient price variations across firms that arise due to the presence of nominal rigidities. In practice, however, this is not the only channel through which monetary policy matters. Recent research has re-affirmed the importance of unanticipated changes in inflation for agents who enter into nominal contracts (see e.g. Doepke and Schneider, 2006). The key mechanism in these models is that unanticipated inflation erodes the real value of nominal debt, leading to redistribution from lenders to borrowers. Even if prices are fully flexible, the choice of monetary policy regime is non-trivial in these circumstances. However, relatively little is known about the merits of inflation targeting and other policy regimes in this context.[1] This paper investigates this issue by assessing the impact of nominal GDP targeting on the tax burden, as measured by the level and volatility of distortionary taxes faced by households.

A model is presented in which monetary policy matters for taxes due to the fact that unanticipated inflation has real wealth effects. The model consists of overlapping generations who work, consume and save optimally over the life cycle. Saving consists of endogenous accumulation of physical capital, government debt and money. Taxes are distortionary because consumption expenditure is taxed at a proportional rate. The main mechanism in the

[1] Exceptions include Meh, Rios-Rull and Terajima (2010), Koenig (2013) and Sheedy (2014).

model is that unanticipated inflation matters for the real debt burden of the government – and hence the path of distortionary taxes – because the government issues nominal debt.[2] Since nominal GDP targeting leads to a *countercyclical* price level, the path of taxes differs under inflation and nominal GDP targeting.[3] The main aim of this paper is to investigate numerically how the tax burden differs under inflation targeting and nominal GDP targeting due to the implications of these regimes for average taxes and tax volatility.

The main findings are as follows. There are differences in both the level and volatility of taxes under inflation and nominal GDP targeting. Under nominal GDP targeting, taxes are higher on average but less volatile. In other words, taxes are smoothed more effectively under nominal GDP targeting, but at a higher average level. The higher level of taxes is driven by the fact that nominal GDP targeting implies a *countercyclical price level*. When the price level is countercyclical, bondholders are hit with surprise inflation at times when their income is low. Since risk-averse agents must be compensated for this increase in risk, average government borrowing costs are higher under nominal GDP targeting,[4] which in turn raises the level of taxes needed to finance government expenditure. At the same time, taxes are less volatile under nominal GDP targeting (except at very high indexation shares). This is because if the price level is countercyclical, government liabilities are eroded (inflated) in real terms at times when tax revenue is low (high) due to falling (rising) output. As a result, tax rates need to vary less in response to output fluctuations under a nominal GDP targeting regime.

When the tax burden is represented by a quadratic loss function, the impact of nominal GDP targeting on the expected tax burden depends on whether the mean effect (higher average taxes) outweighs the volatility effect (lower tax volatility). Under the baseline calibration, the expected tax burden is lower under nominal GDP targeting for a large range of indexation shares, because tax volatility is lowered sufficiently relative to inflation targeting to offset the impact of higher mean taxes. The exception occurs at indexation shares close to 100%, for which the expected tax burden becomes higher under nominal GDP targeting than inflation targeting. This is because tax volatility rises as the share of indexed debt is increased, eventually exceeding the level under inflation targeting at very high indexation shares. In addition, while the expected tax burden is minimized under inflation targeting by issuing only indexed debt, nominal GDP targeting minimizes the tax burden when nominal and indexed debt are issued.[5] These conclusions are fairly robust, but sensitivity analysis indicates that the implications of nominal GDP targeting and inflation targeting for the tax burden depend crucially on risk aversion, productivity persistence and the assumed tax burden loss function.

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[2] In practice, most government debt is nominal. For instance, only around 25% of UK government debt is indexed debt (DMO, 2015). The equivalent figure in Canada is 8% (Department of Finance Canada, 2016), while in the US less 10% of government debt is indexed to inflation (US Treasury, 2016).

[3] Nominal GDP targeting is modelled as nominal GDP *level* targeting in the numerical analysis that follows. However, similar results are obtained for the case of nominal GDP growth targeting; see Section 5.4.

[4] The higher level of borrowing costs under nominal GDP targeting is shown by an increase in the inflation risk premium (see Bekaert and Wang, 2010 for survey). Here, the inflation risk premium is defined as the difference between the expected real returns on nominal and indexed debt. A formal definition is given in the Appendix.

[5] The ‘corner solution’ under inflation targeting is related to the fact that both mean and variance of taxes fall as the share of indexed debt is increased. This is overturned under nominal GDP targeting because the price level is countercyclical, which enables tax smoothing by issuing some nominal debt. Section 4 provides more detail.

In particular, high levels of risk aversion or productivity persistence overturn the result that nominal GDP targeting lowers the expected tax burden, as does a loss function which places a relatively low weight on tax volatility as compared to the case of a quadratic loss function.

The paper is related to two main strands of literature. First, there is a past literature on nominal GDP targeting. Formal analyses include Bean (1983), Bradley and Jansen (1989) Hall and Mankiw (1993), and Jensen (2002), amongst others. These papers highlight circumstances in which nominal GDP targeting can be expected to raise social welfare.[6] More recently, Billi (2013) has shown that nominal GDP targeting may be beneficial in the presence of the zero lower bound on nominal interest rates. A somewhat different issue is studied by Koenig (2013) and Sheedy (2014). They show that if *private* financial contracts are specified in nominal terms, nominal GDP targeting will redistribute income from borrowers to lenders. Due to this risk-sharing mechanism, nominal GDP targeting is able to replicate the efficient allocation that would result in the presence of complete financial markets. The analysis here contributes to the nominal contracting part of the literature by studying the implications of nominal GDP targeting for taxes. This contrasts with previous work in the literature, which has studied nominal GDP targeting in the presence of private nominal debt and wage contracts, but not government debt.[7]

The paper is also related to literature on the redistributive effects of inflation through changes in the real value of nominal debt. In a seminal paper, Doepke and Schneider (2006a,b) show that US households have substantial net nominal positions and that the redistributive effects of unanticipated inflation through this channel are quantitatively significant. In particular, a moderate episode of unanticipated inflation implies substantial wealth losses for older agents, the main holders of government debt.[8] Similar results are shown for Canada in Meh and Terajima (2011) and for the Euro Area in Adam and Zhu (2015). Theoretical analyses of the redistributive effects of inflation through this channel include Champ and Freeman (1990), Doepke and Schnieder (2006c), and Meh, Rios-Rull and Terajima (2010). Like the present paper, these studies use an overlapping generations model. The novel feature here is that monetary policy affects *average* government borrowing costs – and hence average taxes – because the effects of *inflation risk* are taken into account. This is crucial since nominal GDP targeting raises inflation risk relative to inflation targeting, so that higher taxes are necessary to fund government expenditure.

As noted by Doepke and Schneider (2006c) and Meh, Rios-Rull and Terajima (2010), a crucial issue that arises in the context of redistribution through unanticipated inflation is how

[6] With the exception of Jensen (2002), these papers focus on nominal GDP *level* targeting (as here). Since no central bank has adopted this regime in practice, previous studies have been theory-based rather than empirical.
[7] Nominal wage contracts (resp. nominal debt contracts) are central in the results of Bean (1983), Bradley and Jansen (1989) (resp. Koenig (2013), Sheedy (2014)). None of these papers focus on government debt.
[8] Doepke and Schneider (2006a) report that, in the baseline year of 1989, the net nominal position in bonds of US households above the age of 56 ranged from 12.4 to 16.4 percent. Consequently, following a moderate episode of unanticipated inflation "a coalition of rich and old households loses, in present value terms, between 5.7 and 15.2 percent of GDP" (p. 1071). In Doepke and Schneider (2006b), where the foreign sector is excluded, this is updated to: "A coalition of relatively old households loses between 7 and 18 percent of GDP." (p. 500).

any windfalls (losses) are used (funded) by the government. In the model in this paper, government expenditure is exogenous, so windfalls accruing to the government when inflation is unexpectedly high are partly redistributed back to households via lower taxes. The assumption that taxes and government debt take the burden of adjustment following shocks is sensible for a long run analysis (as here) for two reasons. First, both total government outlays (which includes debt interest repayments) and total tax revenue have been rather volatile over time as a share of GDP, whereas government expenditure has been more stable.[9] Second, it is difficult to gain political support for sustained changes in government expenditure, so economists have been active in investigating other mechanisms open to fiscal policymakers.[10]

The remainder of the paper proceeds as follows. Section 2 presents the model. Section 3 provides a description of monetary policy under inflation and nominal GDP targeting. In Section 4, the model is calibrated and the main results are reported. This is followed by a discussion of robustness and sensitivity analysis in Section 5. Finally, Section 6 concludes.

2 Model

The model is a version of Diamond's (1965) overlapping generations model where the young save for old age using capital and government bonds. It contains three sectors: a household sector which saves and supplies labour; a government sector which issues debt, levies taxes and spends; and a firm sector which produces output using capital and labour.[11] This section describes each sector, the competitive equilibrium, and how the tax burden is measured.

2.1 Households

The model consists of a small open economy with an infinitely-lived government and overlapping generations of representative households (of size 1) that live for two periods. Consumption when young (old) is denoted c_1 (c_2). Each generation supplies labour, l, when young. Consumption is taxed at rate τ_c. When young, agents consume out of their labour income $w.l$ and save in capital, k (which depreciates fully within a period); nominal government bonds, b^n; indexed government bonds, b^i; and real money balances, m.[12] When old, agents retire and consume their remaining wealth, leaving no bequests.

Capital earns a risky real return r_k. Nominal bonds pay a riskless gross nominal interest rate R on maturity. The *ex post* real return on nominal bonds is given by $r_{t+1,n} = R_t/\Pi_{t+1}$, where $\Pi_{t+1} \equiv P_{t+1}/P_t$ is inflation, and P_t is the aggregate price level. Indexed bonds pay a riskless real return r_f which coincides, due to arbitrage, with the *constant world real interest rate*, r^*.[13]

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[9] This is related to the fact that the expenditure shares of consumption and investment have been fairly stable. See e.g. the World Bank database at: http://data.worldbank.org/data-catalog/world-development-indicators.
[10] For instance, Aizenman and Marion (2011) assess the option of using inflation to erode the US government debt. There is also a substantial normative literature on optimal taxation in which government expenditure is typically treated as exogenous (see Ljungqvist and Sargent, Chapter 12).
[11] In Diamond (1965), labour supply is inelastic. Endogenous labour supply is considered here since this channel will affect the response of output to shocks, which is potentially important given that nominal GDP targeting responds explicitly to fluctuations in real GDP.
[12] The assumption of full depreciation of capital is reasonable as each period is interpreted as lasting 30 years.
[13] Demand for bonds is fully satisfied by the government. This assumption does not affect the conclusions.

Money pays a real return $r_m = 1/\Pi$. Following Artus (1995), demand for money arises from the requirement that money holdings be at least a fraction θ of consumption expenditure by the young, so that $m_t \leq \theta(1+\tau_{c,t})c_{1,t}$. Taxes enter here because consumption is taxed at rate τ_c. The above constraint will hold with equality if $R_t > 1$ for all t, which is assumed to hold.[14]

Hence we have that

$$m_t = \theta(1+\tau_{c,t})c_{1,t}, \qquad \forall t \tag{1}$$

The nominal money supply in period t is denoted M_t and follows a fixed rule. Changes in the money stock are accomplished by lump-sum money transfers to the old of $S_t = M_t - M_{t-1}$, which are taken as given by households. Real money balances in period t are given by $m_t = M_t/P_t$. The real money transfer received by the old is defined as $s_t = S_t/P_t$.

A young agent born at date t solves the following problem:

$$\max_{\{c_{1,t}, c_{2,t+1}, l_t, m_t, k_{t+1}, b^i_{t+1}, b^n_{t+1}\}} \quad U(c_{1,t}) + \beta E_t U(c_{2,t+1}) - U(l_t)$$

s.t.
$$(1+\tau_{c,t})c_{1,t} = w_t l_t - k_{t+1} - b^i_{t+1} - b^n_{t+1} - m_t \tag{2}$$
$$(1+\tau_{c,t+1})c_{2,t+1} = r_{t+1,k}k_{t+1} + r_{t,f}b^i_{t+1} + r_{t+1,n}b^n_{t+1} + r_{t+1,m}m_t + s_{t+1} \tag{3}$$

where $U(c_{j,t}) = \dfrac{c_{j,t}^{1-\gamma}}{1-\gamma}$, $U(l_t) = \dfrac{l_t^{1+\eta}}{1+\eta}$ and $\beta > 0$ is the private discount factor.

The first-order conditions are:

$$\frac{U_c(c_{1,t})}{(1+\tau_{c,t})} = \frac{U_l(l_t)}{w_t} + \theta\mu_t \qquad \text{for labour supply, } l \tag{4}$$

$$\frac{U_c(c_{1,t})}{(1+\tau_{c,t})} = \beta E_t\left[\frac{U_c(c_{2,t+1})}{(1+\tau_{c,t+1})} r_{t+1,k}\right] + \theta\mu_t \qquad \text{for capital, } k \tag{5}$$

$$\frac{U_c(c_{1,t})}{(1+\tau_{c,t})} = \beta E_t\left[\frac{U_c(c_{2,t+1})}{(1+\tau_{c,t+1})} r_{t+1,n}\right] + \theta\mu_t \qquad \text{for nominal bonds, } b^n \tag{6}$$

$$\frac{U_c(c_{1,t})}{(1+\tau_{c,t})} = \beta\, r_{t,f} E_t\left[\frac{U_c(c_{2,t+1})}{(1+\tau_{c,t+1})}\right] + \theta\mu_t \qquad \text{for indexed bonds, } b^i \tag{7}$$

$$\frac{U_c(c_{1,t})}{(1+\tau_{c,t})} = \beta E_t\left[\frac{U_c(c_{2,t+1})}{(1+\tau_{c,t+1})\Pi_{t+1}}\right] + (1+\theta)\mu_t \qquad \text{for money holdings, } m \tag{8}$$

where $U_c(c_{j,t}) = c_{j,t}^{-\gamma}$, $U_l(l_t) = l_t^{\eta}$, and μ_t is the Lagrange multiplier on Equation (1).

[14] This condition is derived in Section A of the Supplementary Appendix and was satisfied in all numerical simulations reported in this paper. Crettez, Michel and Wigniolle (1999) survey the ways money has been introduced in overlapping generations models. Money in the utility function has similar results (see Section 5.1).

2.2 Firms

The production sector consists of a representative firm that produces output using a Cobb-Douglas production function: $y = Ak^{\alpha}l^{1-\alpha}$. The capital income share in output is equal to α and the labour share is $1-\alpha$. The firm hires capital and labour in competitive markets to maximise current period profits. Total factor productivity, A, is stochastic and follows an AR(1) in logs:

$$\ln(A_t) = \rho_A \ln(A_{t-1}) + e_t$$

where e_t is an IID-normal innovation with mean zero and standard deviation σ_e.

The real wage and the return on capital are given by

$$w_t = (1-\alpha)A_t(k_t/l_t)^{\alpha} \tag{9}$$

$$r_{t,k} = \alpha y_t/k_t = \alpha A_t(k_t/l_t)^{\alpha-1} \tag{10}$$

2.3 Government

The government issues debt in the amount demanded by households and sets consumption taxes τ_c to cover fixed expenditure $g^* > 0$ plus interest payments on debt net of new issuance.[15] The government budget constraint is given by:[16]

$$\begin{aligned}\tau_t(c_{1,t}+c_{2,t}) &= g^* + r_{t-1,f}b_t^i - b_{t+1}^i + (R_{t-1}/\Pi_t)b_t^n - b_{t+1}^n \\ &= g^* + [v_t r_{t-1,f} + (1-v_t)(R_{t-1}/\Pi_t)]b_t - b_{t+1}\end{aligned} \tag{11}$$

where $b = b^n + b^i$ is total government debt issued and $v \equiv b^i/b$ is the share of indexed debt.

In what follows, we consider equilibria where the share of indexed debt is held constant. Hence, the supply of nominal debt is given by $(1-v)b_t$, and the supply of indexed debt is vb_t, as in Hatcher (2014). Given this portfolio allocation, the total demand for government debt is determined endogenously by the first-order conditions. The supply of government debt is set equal to this demand, which ensures market-clearing for each type of debt.

2.4 Equilibrium

Since capital depreciates fully within a period, investment in period is given by $i_t = k_{t+1}$.

Definition of equilibrium:[17]

A set of allocations and prices $\left\{c_{1,t}, c_{2,t}, b_t^{i,d}, b_t^{i,s}, b_t^{n,d}, b_t^{n,s}, k_{t,} \ l_t, m_t^d, m_t^s, \tau_{c,t}, R_t, r_{t,f}, M_t, P_t\right\}_{t=0}^{\infty}$ with the following properties:

[15] The key assumption is that government expenditure is exogenous. Since government expenditure fluctuations do not add any insights, g is held fixed in order to reduce the number of parameters and shocks in the model.

[16] Note that the money supply does not appear in the government budget constraint because changes in the money stock finance lump-sum transfers to the old.

[17] Note that d and s superscripts are introduced in this section to denote demand and supply values. These superscripts are omitted in other sections of the paper in order to reduce on notation.

(1) Allocations $\left\{c_{1,t}, c_{2,t+1}, l_t, b_{t+1}^{i,d}, b_{t+1}^{n,d}, k_{t+1}, m_t^d\right\}$ solve the utility maximisation problem of the young born at date *t*;

(2) The goods, bond and money markets clear:

$$y_t = c_{1,t} + c_{2,t} + k_{t+1} + g^*$$
$$b_t^{i,d} = b_t^{i,s}, \quad b_t^{n,d} = b_t^{n,s},$$
$$m_t^d = m_t^s = M_t / P_t$$

(3) The government budget constraint, (11), holds and the share of indexed debt *v* is constant;

(4) Factors of production are paid their marginal products, (9) and (10), and the real interest rate on indexed bonds equals the constant world interest rate r^*;

(5) The cash constraint on money holdings, (1), holds with equality: $m_t = \theta(1+\tau_{c,t})c_{1,t}$;

(6) Given $\{k_0, b_0, M_0, R_{-1}, r_{-1,f} = r^*\}$, $c_{2,0} = (1+\tau_{c,0})^{-1}[r_{0,k}k_0 + (vr_{-1,f} + (1-v)R_{-1}/\Pi_0)b_0 + M_0/P_0]$ is consumed by the initial old.

2.5 Measuring the tax burden

To measure the tax burden, an ad hoc quadratic loss function is considered. In particular, it assumed that the period loss function is $TB_t = (\tau_{c,t}/\tau_c)^2$, where τ_c is the steady-state consumption tax rate.[18] Since the loss will vary over time with shocks that hit the economy, the numerical analysis below focuses on the long run average tax burden by calculating the unconditional expectation of the period loss, $E[TB]$. We consider a quadratic loss function because it penalises both higher taxes and tax volatility.[19] In particular, an increase in mean taxes will raise the expected tax burden, as will a mean-preserving increase in tax volatility.

Formally, the burden-minimising debt share solves the problem:

$$\min_{v\in[0,1]} E[TB_t] = \frac{1}{\tau_c^2} E\left[\tau_{c,t}^2\right] \qquad (12)$$

s.t. (1)-(11), the equilibrium conditions (see Section 2.4), and

$$\Pi_t = \begin{cases} \Pi_t^{IT} & \text{under inflation targeting (IT)} \\ \Pi_t^{NGDP} & \text{under nominal GDP targeting (NGDP)} \end{cases}$$

The debt share that solves the above problem is computed numerically. To do so, the model was solved using a second-order approximation in Dynare (see Adjemian et al., 2011). In

[18] Dividing by the steady-state tax burden, $(\tau_c)^2$, gives the tax burden a meaningful interpretation since it is measured relative to its constant steady-state value.

[19] Tax volatility matters when taxes are distortionary (as here). In particular, if tax distortions increase at the margin as taxes rise, then it is optimal, under certain circumstances, to keep taxes constant. This the 'tax-smoothing' motive identified by Barro (1979).

particular, the unconditional expectation of the tax burden was computed for a large number of discrete debt shares in the interval [0,1] by looping over the parameter v in small steps.

To understand the numerical results that follow, it is instructive to consider a second-order approximation of the expected tax burden around the point $\tau_{c,t} = E[\tau_{c,t}]$:[20]

$$E[TB_t] \approx \underbrace{(E[\tau_{c,t} / \tau_c])^2}_{\text{Mean effect}} + \underbrace{\frac{1}{\tau_c^2} \text{var}[\tau_{c,t}]}_{\text{Volatility effect}} \tag{13}$$

This expression shows that the expected tax burden increases with both mean taxes and tax volatility. These moments are therefore reported in the numerical analysis that follows. In addition, the decomposition of Equation (13) into a mean effect and a volatility effect is used to shed light on the relative importance of these two components for the tax burden.

3 Monetary policy

Money market clearing requires that $M_t = P_t\, m_t$, where m_t is given by Equation (1). Hence, inflation is $\Pi_t = P_t / P_{t\text{-}1} = (M_t / M_{t\text{-}1}).(m_{t\text{-}1} / m_t)$. However, the central bank has only imperfect control of the price level because the money supply is subject to a control error $\exp(\varepsilon_t)$, where ε_t is IID-normal with mean zero and standard deviation σ_ε.[21] Due to these money supply shocks, the central bank will not exactly meet its target for inflation or nominal GDP.

Under inflation targeting, 'bygones are bygones' so the nominal money supply evolves according to the following rule:

$$M_t = M_{t-1} \Pi_t^* (m_t / m_{t-1}) \exp(\varepsilon_t) \tag{14}$$

where Π_t^* is the desired inflation rate.

The desired inflation rate is given by the inflation target, Π^*. Hence, by Equation (14) and money market clearing, inflation is given by

$$\Pi_t^{IT} = \Pi^* \exp(\varepsilon_t) \tag{15}$$

Notice that inflation would be exactly equal to target in the absence of money supply shocks.

By contrast, *level targeting* regimes call for past deviations from target to be offset in the next period. As a result, the money supply rule under nominal GDP (level) targeting includes both the current control error and a response to undo the past control error $\exp(\varepsilon_{t\text{-}1})$:

$$M_t = M_{t-1} (P_t^* / P_{t-1}^*)(m_t / m_{t-1}) \exp(\varepsilon_t) / \exp(\varepsilon_{t-1}) \tag{16}$$

where P^* is the desired price level so that the nominal GDP target would be met each period.

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[20] Note that the middle term in the second-order expansion drops out because $E[\tau_{c,t} - E[\tau_{c,t}]] = 0$.

[21] As discussed in Section 5.3, adding persistence in money supply shocks does not add any additional insights.

The desired price level P_t^* satisfies $P_t^* y_t = (Py^*)_t$, where $(Py^*)_t = [\Pi^*(1+\Delta y^*)]^t$ is the nominal GDP level target in period t, and target output growth is assumed to be zero (i.e. $\Delta y^* = 0$).[22] As a result, desired inflation is $(y_{t-1}/ y_t).(Py^*)_t /(Py^*)_{t-1} = \Pi^* y_{t-1}/ y_t$. Hence, by Equation (16) and money market clearing, inflation under nominal GDP targeting is given by:[23]

$$\Pi_t^{NGDP} = \Pi^*[y_{t-1} / y_t]\exp(\varepsilon_t)/\exp(\varepsilon_{t-1}) \quad (17)$$

Nominal GDP targeting implies a *countercyclical* price level: it makes the price level and output *negatively correlated*. By contrast, there is no relationship between the price level and output under inflation targeting, so that the only source of inflation risk is the control error ε_t.

The inflation variances under the two regimes are related as follows:

$$\text{var}_{t-1}(\ln \Pi_t^{NGDP}) = \text{var}_{t-1}(\ln \Pi_t^{IT}) + \sigma^2_{y,NGDP} \quad (18)$$

where the second term on the right hand side is the conditional variance of log output under nominal GDP targeting.

Intuitively, inflation is more volatile under nominal GDP targeting because it responds to fluctuations in output, which are primarily driven by productivity shocks. In the numerical analysis that follows, it is shown that this relationship between productivity shocks and inflation variations has implications for both the level and variability of distortionary taxes.

4 Results

The model was solved using a second-order perturbation in Dynare (Adjemian et al., 2011). The tax burden-minimizing indexation share was computed as described in Section 2.5. This section describes the baseline calibration and steady-state before turning to the main results.

4.1 Calibration and model solution

4.1.1 Calibration

The model is calibrated for the UK economy. In particular, the parameters of the model are chosen to roughly match key ratios in the data. Since these ratios depend upon several different parameters, the calibration uses parameter values which are plausible *and* give good overall performance against target ratios.[24] For the purpose of calibration, each period in the model is assumed to last 30 years. The baseline calibration is listed in Table 1.

The parameter α was set at 0.30, implying a share of capital income in GDP of 30% and a labour income share of 70%, which is fairly standard. The private discount factor β is set

[22] It is assumed, without loss of generality, that the initial nominal GDP target, $(Py^*)_0$, is equal to one.
[23] The response of inflation to the lagged policy error ε_{t-1} reflects the fact that past deviations from the nominal GDP target are offset under nominal GDP *level* targeting. Under nominal GDP *growth* targeting there is no response to the lagged policy error but very similar results are obtained; see Section 5.4.
[24] In some models, key ratios can be pinned down by a single parameter so that calibrated values can be set to match target ratios exactly. The model here does not have this property.

equal to 0.65, which is equivalent to an annual value of 0.979 under the assumption that each period lasts 30 years.[25] The coefficient of relative risk aversion, γ, is set equal to 2, which is a standard value that lies between the calibrations used in the business cycle and asset pricing literatures. The inverse Frisch elasticity η was set equal to 3, which lies in the mid-range of estimates used by the Congressional Budget Office of the United States (see Reichling and Whalen, 2012). The parameter θ was set at 0.10. This implies that the demand for real money balances is 10% of consumption when young, which helps the model to get close to the UK ratio of notes and coins to GDP of 3% (see Fish and Whymark, 2015).

Table 1 – Baseline calibrated values

Parameter	Value
Capital share in output, α	0.30
Private discount factor, β	0.65
Coef. of relative risk aversion, γ	2
Inverse Frisch elasticity, η	3
Cash constraint parameter, θ	0.10
Trend inflation, Π^*	1.81
World real interest rate, r^*	1.9
Real government expenditure, g^*	0.14
Share of indexed debt, v	0.25
Productivity persistence, ρ_A	0.50
Std.(prod. innov.), σ_e	0.05
Std.(money innov.), σ_ε	0.05

Trend inflation is set at $\Pi^* = 1.81$. This amounts to annual inflation of 2% a year, consistent with the UK inflation target for the Consumer Prices Index (CPI). The world real interest rate r^* is set at 1.9, which implies an annualised real interest rate of 2.2%. This is set slightly below the average UK estimate of 2.9% from 1965 to 2005 (see Mills, 2008), because matching a real rate this high would give an investment-GDP ratio somewhat lower than in the data (see Table 2). Given the trend inflation rate of 2% per annum, the implied nominal interest rate is 4.2% per annum. The parameter g^* was set at 0.14 because, in conjunction with the other parameters, this gives a government expenditure share of around 20 per cent, which is similar to the ratio in UK data over the past decade.[26] For the purpose of calibration, the share of indexed government debt, v, was set at 0.25, which matches the current UK share of 25% reported in DMO (2015). The indexation share has no impact on deterministic steady-state but does affect the stochastic steady state and expected tax burden.

The parameter ρ_A was set at 0.50 since there is no convincing evidence that productivity is persistent over generational horizons, as noted by Olovsson (2010, Footnote 21). The productivity innovation standard deviation was set at $\sigma_e = 0.05$, which is similar to the

[25] In sensitivity analysis, alternative values of 0.45 and 0.85 are considered.
[26] See the World Development Indicators published by the World Bank. The expenditure shares reported in Table 2 are all taken from this database.

calibration in Hatcher (2014) in an overlapping generations model where each period is 20 years. The standard deviation of the money supply innovation was also set at $\sigma_\varepsilon = 0.05$. With this calibration, the standard deviation of inflation is 9%, whereas the 20-year standard deviation of UK inflation from 1988 to 2015 is 7%, based on overlapping 20-year sections of the Consumer Prices Index (CPI) published by the Office for National Statistics (ONS).[27]

4.1.2 Model solution

Table 2 reports the deterministic and stochastic steady states of the model under the baseline calibration and inflation targeting. The model does fairly well against target ratios. The consumption expenditure share is 0.65, the investment share is 0.16, and the government expenditure share is 0.19. These values are close to the average national expenditure shares in the UK over the years 2004-2014 in World Bank data (see Table 2, Column 3). The tax revenue target was set at 0.26 based on the average ratio of tax revenue to GDP in the UK over the period 2004-2013 in World Bank data. The calibrated model matches this target.

For government debt, the target ratio was set at 0.11, which matches the ratio of long-term debt to GDP implied by ONS and DMO data.[28] On this score, the model gives a ratio to GDP of 0.08. Hence, the model undershoots the target ratio for government debt somewhat, albeit that the difference is not dramatic. Finally, the calibrated model slightly overshoots the target ratio of currency to GDP. As discussed below, money plays an important role in the results.[29]

Table 2 – Model solution versus key ratios (baseline calibration)

Model ratio	Definition in data	Target	Deterministic	Stochastic	Notes
$(c_1 + c_2)/y$	Consumption/GNE	0.63	0.65	0.65	UK data: WB
i/y	Investment/GNE	0.17	0.16	0.16	UK data: WB
g/y	Govt. Expenditure/GNE	0.20	0.19	0.19	UK data: WB
$\tau_c(c_1 + c_2)/y$	Tax Revenue/GDP	0.26	0.26	0.26	UK data: WB
b/y	Long-Term Govt. Debt/GDP	0.11	0.08	0.08	UK data: ONS and DMO
m/y	Notes and Coins/GDP	0.03	0.04	0.04	UK data: ONS

Notes: ONS = Office National Statistics; WB = World Bank; DMO = Debt Management Office.

Consistent with the findings from the long run empirical literature on the effect of inflation on real variables, changes in the steady-state money supply growth rate (and trend inflation)

[27] Twenty-year sections were used for calibration in order to increase the number of data points available.
[28] The debt-GDP ratio averaged 48% from 2000-2009 (World Bank) and over the same period the average share of long-term nominal government debt was 22% (see DMO, 2015). Combining these two figures gives the target debt-GDP ratio of 11% of GDP. The DMO classifies gilts as 'long-term' if maturity exceeds 15 years.
[29] The author is grateful to a referee for pointing out the importance of moving away from a 'cashless economy'.

have a very small quantitative impact on real variables.[30] In particular, an increase in the steady-state inflation rate slightly lowers the demand for real money balances, the capital stock and labour supply (and hence output), but marginally raises government debt and taxes.

Impulse responses to productivity and money supply shocks are reported in Section B of the Supplementary Appendix. A positive productivity shock raises output, along with consumption when young and old due to the positive income effect. Consequently, the tax base (=aggregate consumption) is rising, so that the tax rate can be lowered whilst maintaining government expenditure. This is exactly what happens under inflation targeting. However, under nominal GDP targeting, taxes respond less. In particular, because an increase in output requires below-target inflation in order to meet the nominal GDP target, the real debt burden of the government rises. Since a rise in the tax base now coincides with an increase in government liabilities, the reduction in taxes needed to maintain government expenditure is smaller than under inflation targeting. Similarly, a negative productivity shock raises taxes less under nominal GDP targeting than it does under inflation targeting.

A positive money supply shock raises inflation on impact, lowering the real value of nominal debt held by the old. This lowers consumption by the old. At the same time, the tax rate falls due to the reduction in government liabilities. Due to the fall in the tax rate, consumption by the young rises, as does saving in capital and bonds. The induced saving is larger under inflation targeting because expected future inflation is equal to the inflation target, so that the expected real return on nominal debt is stable. By contrast nominal GDP targeting creates the expectation of below-target inflation in the future, which makes saving through money relatively more attractive.[31] However, the larger response of government debt under inflation targeting hinges on the relatively low share of indexed debt under the baseline calibration. Government debt and capital respond by more under nominal GDP targeting once most debt is indexed because there is then substitution from capital and bonds to money. The additional volatility of debt under nominal GDP targeting is important for understanding how inflation and nominal GDP targeting compare at high indexation shares, as discussed below.

4.2 Baseline results

The model was solved using a second-order perturbation in Dynare (Adjemian et al., 2011) to obtain approximate theoretical moments. The debt share that minimises the expected tax burden was computed as described in Section 2.5. As described there, the expected tax burden is calculated relative to its deterministic steady state value, $(\tau_c)^2$, which is identical under inflation and nominal GDP targeting and constant as the indexation share is varied.

Figure 1 reports the expected tax burden under inflation and nominal GDP targeting and how it changes as the share of indexed debt is varied from 0 to 1 (i.e. 0-100%). Figure 2 provides

[30] Bullard and Keating (1995) found that inflation has little or no long run effect on output in a large sample of developed and developing countries. Similarly, Crosby and Otto (2000) conclude that inflation has no long run impact on the capital stock. Sidrauski (1967) developed a model consistent with this finding.

[31] Inflation undershoots the inflation target in later periods in order restore nominal GDP to its target path. This requires offsetting the initial impact on inflation and lowering inflation below target when output is increasing.

further detail by decomposing the expected tax burden into mean and volatility effects (see Equation (13)), while Figure 3 highlights the role of key variables that drive the results.

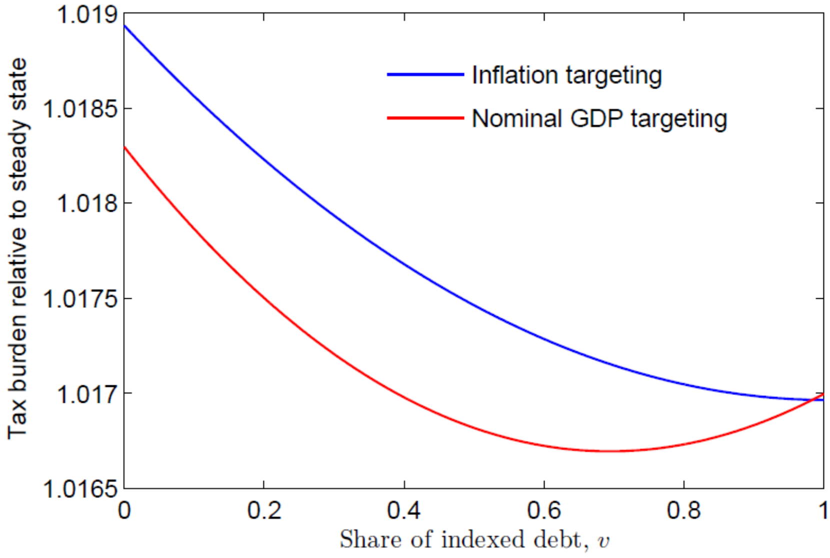

Fig 1 – Expected tax burden and indexation. Figure plots $E[TB]$ as share of indexed debt varies from 0 to 1.

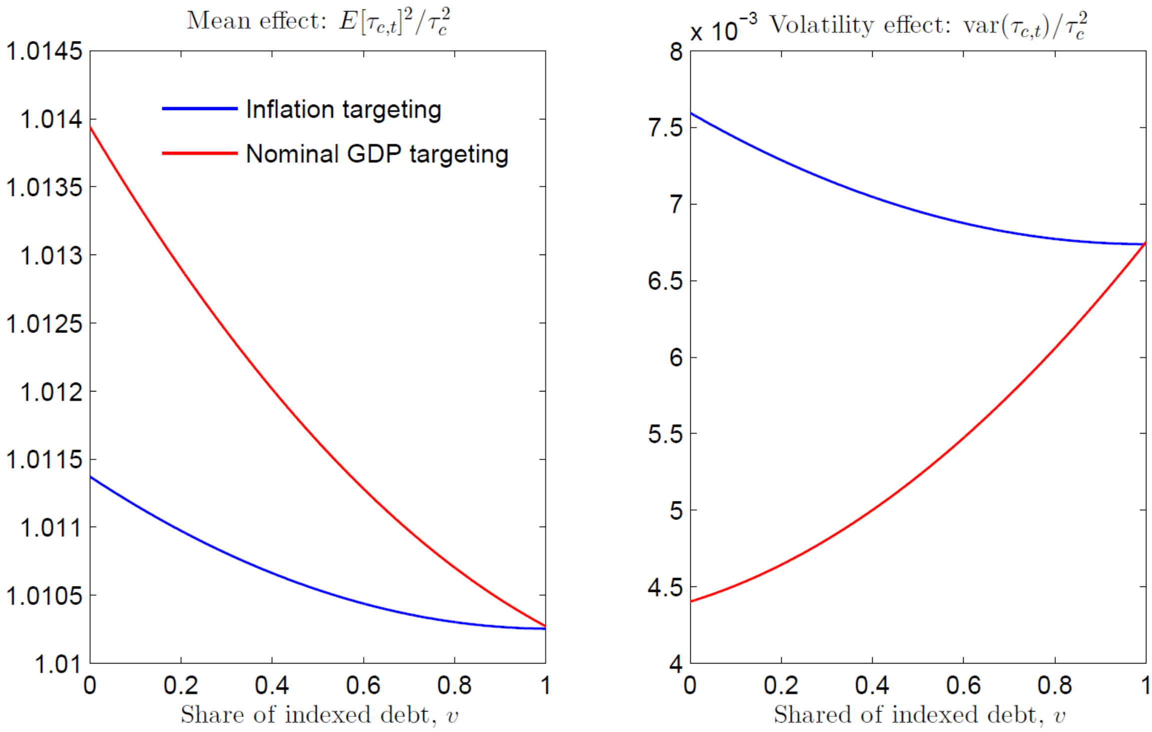

Fig 2 – Decomposition of expected tax burden into mean and volatility effect. Figure plots the mean effect and the volatility effect as the share of indexed debt is varied from 0 to 1. The sum of the mean and volatility effects equals expected tax burden plotted in Fig 1.

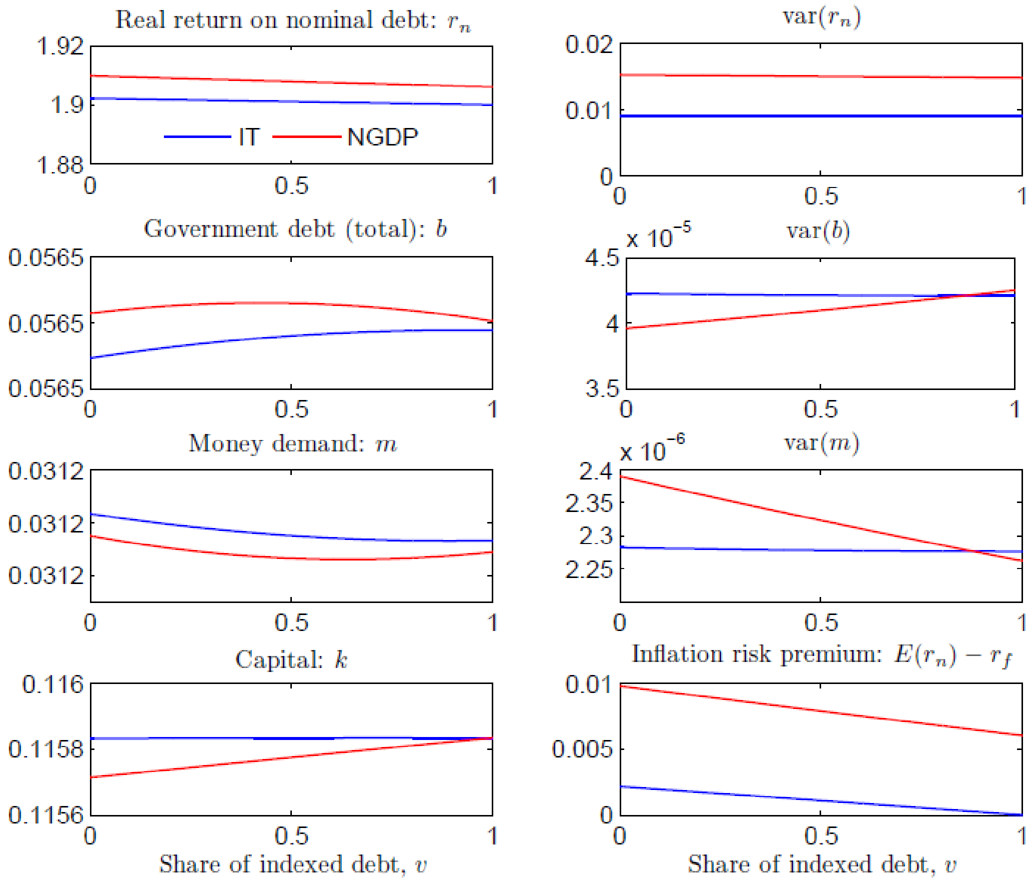

Fig 3 – Real variables and indexation. Figure plots the unconditional moments of variables under inflation targeting and nominal GDP targeting as the share of indexed debt is varied from 0 to 1.

4.2.1 Inflation targeting

Under inflation targeting the expected tax burden falls as the share of indexed debt is increased. This monotonic relationship stems from the fact that both mean taxes and tax volatility decline as the indexation share is increased (see Figure 2). Mean taxes fall because there is a positive inflation risk premium (see Figure 3, bottom row), so that nominal debt is more costly to repay than indexed debt and therefore requires higher average taxes in order to maintain the same level of government expenditure. Issuing more indexed debt allows the government to avoid this cost. In addition to this, issuance of indexed debt lowers the inflation risk premium itself because it means that a smaller fraction of asset portfolios is exposed to inflation risk, so that less risk compensation is demanded by households.[32]

The volatility of taxes falls as the share of indexed debt is increased because the real return on nominal debt is volatile due to inflation risk (see Figure 3, top row), unlike the real return on indexed debt. As the indexation share is increased, the real debt burden of the government becomes more stable, so that less variation in the tax rate is necessary in order to fund government expenditure. This has the knock-on effect of making expected lifetime

[32] Under inflation targeting, the inflation risk premium falls as the share of indexed debt is increased and is zero when only indexed debt is issued. The Appendix derives an expression for the inflation risk premium and explains this result in detail.

consumption growth more stable, so that the demand for government debt becomes slightly less volatile as indexation is increased (see Figure 3, second row). This contrasts with the case of nominal GDP targeting and is important for understanding the results that follow.

The results in Figure 1 show that the expected tax burden is higher under inflation targeting than nominal GDP targeting for the large majority of indexation shares. The exception is at very high shares of indexed debt. In particular, inflation targeting gives a lower expected tax burden than nominal GDP targeting when the share of indexed debt is 98.4% or higher.[33]

4.2.2. Nominal GDP targeting

Under nominal GDP targeting, the relationship between the expected tax burden and the indexation share is not monotonic as under inflation targeting. The expected tax burden initially falls as the share of indexed debt is increased, but there are sharp increases in the tax burden after a minimum value is reached. The indexation share that minimizes the expected tax burden is 69.4% under the baseline calibration (see Figure 1). Hence, in order to minimize the expected tax burden, around one-third of government debt should be nominal and around two-third indexed, under a nominal GDP targeting regime. This is in stark contrast to inflation targeting, where the expected tax burden is minimized by issuing only indexed debt.

This difference in results can be understood in terms of the mean and volatility effects (see Figure 2). As under inflation targeting, mean taxes fall as the share of indexed debt is increased because the government avoids paying the inflation risk premium (see Figure 3, bottom row). However, mean taxes are higher under nominal GDP targeting. The reason is that nominal GDP targeting makes the price level *countercyclical*. As a result, the old are hit with unanticipated inflation at times when income is low. This exacerbates falls in consumption in response to negative productivity shocks, and therefore raises marginal utility in bad states. Since households require compensation for this increase in risk, mean nominal interest rates rise relative to inflation targeting, raising the expected real return payable on nominal debt (top row, Figure 3), and with it the inflation risk premium.[34] Consequently, repaying nominal debt is more costly under nominal GDP targeting, and mean taxes are higher. If only this effect were present, nominal GDP targeting would unambiguously raise the tax burden relative to inflation targeting. This conclusion applies even if only indexed debt is issued. In that corner case, debt will not be more expensive to repay under nominal GDP targeting (because no nominal debt is actually issued) but taxes nevertheless remain higher because government debt is slightly higher on average (see Figure 3, second row).

In fact, however, the expected tax burden is *lower* under nominal GDP targeting, unless very high indexation shares are reached (see Figure 1). The reason is that the volatility of taxes is minimized when only *nominal* debt is issued, and this difference in volatility is enough to partially offset the effect of higher average taxes. With the mean and volatility effects going

[33] The exact point of intersection depends on the parameterization of the model, as shown in Section 5.3.
[34] The inflation risk premium falls as the share of indexed debt is increased (as under inflation targeting) but remains positive under full indexation of government debt (in contrast to inflation targeting). The Appendix explains the reasons for this difference.

in opposite directions under a nominal GDP targeting regime, the net result is that the expected tax burden is minimized at an interior indexation share. The fact that this occurs at an indexation share of 69.4% indicates that the volatility effect contributes substantially to the overall tax burden. Indeed, for indexation shares below 75% the volatility of taxes is lowered under nominal GDP targeting by more than one-tenth (see Figure 2). However, as the share of indexed debt is increased, the volatility of taxes rises sharply and it eventually overtakes that under inflation targeting at an indexation share close to 100%.

The relatively low volatility of taxes under nominal GDP targeting is due to the mechanism described in Section 4.1.2: in response to productivity shocks, the central bank responds by moving inflation in the opposite direction, so that the price level is countercyclical. As a result, the real debt burden faced by the government falls in periods when output is low and rises in periods when output is high. Because the tax base (= aggregate consumption) also falls when output is low and rises when output is high, the presence of a countercyclical price level ensures that the tax base and debt repayments move in tandem. Consequently, smaller movements in the tax rate are necessary to maintain government expenditure in the face of productivity shocks. The above description brings out the importance of productivity shocks for lower tax volatility under nominal GDP targeting. Indeed, reducing the volatility of productivity innovations relative to the volatility of money supply shocks closes the gap between tax volatility under inflation and nominal GDP targeting and thus raises the indexation share at which the expected tax burden is minimized (see Section 5.4).

It should also be noted that the presence of money and interaction between government debt and money plays an important role in the results. In particular, if money were absent (the case where $\theta = 0$), then the expected tax burden under nominal GDP targeting would be lower for *all* interior indexation shares and exactly equal at the corner solution where all debt is indexed. Similarly, increasing real money holdings by raising θ will increase the range of indexation shares for which the expected tax burden is lower under inflation targeting, albeit that this still only occurs at indexation shares close to 100% (see Section 5.3). The result that nominal GDP targeting raises the expected tax burden at high indexation shares is driven by the fact that government debt volatility rises sharply as indexation is increased and eventually overtakes the level under inflation targeting (see second row, Figure 3). This channel is entirely absent in an economy without money.

The additional volatility comes from the fact that inflation risk is higher under nominal GDP targeting due to the need to respond to fluctuations in real GDP and past money supply shocks (see Equations (17) and (18)). In particular, with inflation being more volatile in response to shocks, there are greater fluctuations in the expected real return on money, which in turn drive volatility in the marginal value of holding money, μ_t. Since this marginal value enters into the first-order conditions for bond holdings, this translates into volatility in the demand for government debt (see the two middle rows of Figure 3). At relatively low indexation shares this volatility remains lower than under inflation targeting. However, this volatility rises as the indexation share is increased because the burden of adjustment falls increasingly upon the quantity of government debt given that the world real interest rate is exogenous.[35] As Figure 3 clearly shows, the volatility of government debt under nominal GDP targeting exceeds that under inflation targeting after an indexation share of around 80% is reached. As the importance of money holdings is increased by raising θ, this point of intersection happens at slightly lower indexation shares, which in turn causes the expected tax burden under nominal GDP targeting to be higher than under inflation targeting for a slightly wider range of indexation shares, as discussed in Section 5.3. Hence, money and the interaction between money and government debt are important for understanding the behaviour of the tax burden. Similar conclusions are reached for the case of money in the utility function (see Section 5.1).

4.3 Discussion and policy implications

The above results indicate that the share of indexed government debt matters for the tax burden and that this relationship is somewhat different under inflation targeting and nominal GDP targeting. Accordingly, the indexation shares that minimize the expected tax burden are very different – 100% indexed debt under inflation targeting, as compared to around one-third nominal debt and two-thirds indexed debt under nominal GDP targeting. These results are interesting in light of the fact that indexation shares in advanced economies are well below 50% (see Campbell et al., 2009 and Footnote 2). It is also of interest that the tax burden depends on the demand for money and government debt. In particular, the results suggest that the tax burden might vary across countries where currency is a small share of GDP (UK, US), and those (Japan, China) where it is relatively more important (Financial Times, 2014; Bank of Japan, 2016).

5 Robustness

This section considers the robustness of the baseline results to the way that money demand is introduced, the specification of the tax burden, and calibration of model parameters. It also compares nominal GDP *growth* targeting with the level-targeting regime of the baseline case.

5.1 Money in the utility function

In the baseline model, demand for money was introduced via a requirement that the young hold real money balances in proportion to their consumption, as in Artus (1995) and Crettez et al. (1999). To test robustness, this section considers the case where money enters into the utility function (Sidrauski, 1967). Examples of overlapping generations models which introduce money in this way include McCallum (1986) and Nikitin and Russell (2003).

With this assumption, the maximization problem of a young agent born at date *t* is now:

$$\max_{\{c_{1,t},\, c_{2,t+1},\, l_t,\, m_t,\, k_{t+1},\, b^i_{t+1},\, b^n_{t+1}\}} \quad U(c_{1,t}) + \beta E_t U(c_{2,t+1}) + \theta_m U(m_t) - U(l_t) \quad \text{s.t. (2), (3)}$$

where $U(m_t) = \dfrac{m_t^{1-\gamma_m}}{1-\gamma_m}$ and θ_m is the relative weight attached to utility from money holdings.

[35] In particular, the total real return on government debt is $vr^* + (1 - v)R_{t-1}/\Pi_t$.

The first-order conditions are now:

$$\frac{U_c(c_{1,t})}{(1+\tau_{c,t})} = \frac{U_l(l_t)}{w_t} \qquad \text{for labour supply, } l$$

$$\frac{U_c(c_{1,t})}{(1+\tau_{c,t})} = \beta E_t\left[\frac{U_c(c_{2,t+1})}{(1+\tau_{c,t+1})} r_{t+1,k}\right] \qquad \text{for capital, } k$$

$$\frac{U_c(c_{1,t})}{(1+\tau_{c,t})} = \beta E_t\left[\frac{U_c(c_{2,t+1})}{(1+\tau_{c,t+1})} r_{t+1,n}\right] \qquad \text{for nominal bonds, } b^n$$

$$\frac{U_c(c_{1,t})}{(1+\tau_{c,t})} = \beta\, r_{t,f}\, E_t\left[\frac{U_c(c_{2,t+1})}{(1+\tau_{c,t+1})}\right] \qquad \text{for indexed bonds, } b^i$$

$$\frac{U_c(c_{1,t})}{(1+\tau_{c,t})} = \theta_m U_m(m_t) + \beta E_t\left[\frac{U_c(c_{2,t+1})}{(1+\tau_{c,t+1})} \Pi_{t+1}^{-1}\right] \qquad \text{for money holdings, } m$$

The relative weight on money holdings in the utility function, θ_m, is set at 0.01 so that real money balances are 4% of GDP as in the baseline model. The utility curvature parameter γ_m is set equal to 2, which implies that utility from consumption and utility from real money balances have the same curvature.[36] All the other parameters are given the same values as in the baseline case (see Table 1). The results are very similar to those from the baseline model and are reported in full in Section C of the Supplementary Appendix. Figure 4 illustrates this similarity by directly comparing the expected tax burden in the two cases.

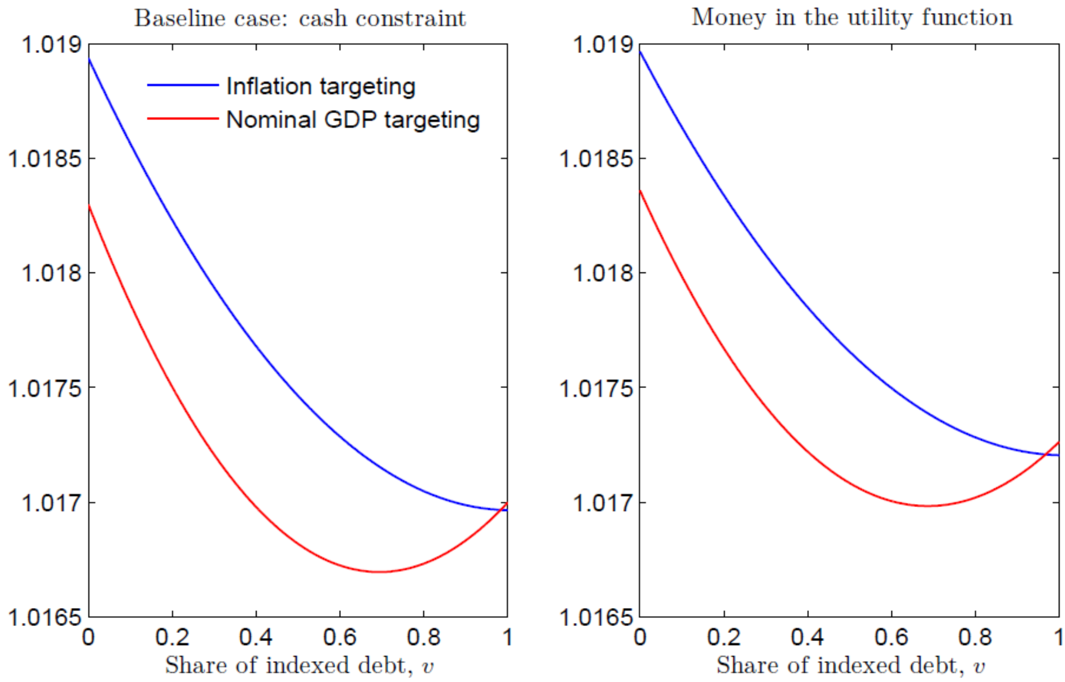

Fig 4 – Expected tax burden and indexation: baseline case vs money in the utility function.
Figure plots $E[TB]$ as the share of indexed debt, v, is varied from 0 to 1.

[36] Experimentation with values in the range 1.5-2.5 was tried but did not make much difference to the results.

Under nominal GDP targeting, the expected tax burden is now minimized at a slightly lower indexation share of 68.6% (as compared to 69.4% in the baseline case) and the expected tax burden is lower under nominal GDP targeting until an indexation share of 97.0% is reached (as compared to 98.4% in the baseline case). This is again driven by the volatility effect: taxes become increasingly volatile under nominal GDP targeting as indexation is increased, and more so than under inflation targeting at a slightly lower indexation share than in the baseline case. This is explained mainly by the fact the government debt volatility now overtakes the level under inflation targeting at a lower indexation share than in the baseline case. This channel would be absent in the cashless case where $\theta_m = 0$. All in all, the results are robust to the way that demand for money is introduced.

5.2 Alternative specifications for the tax burden

The baseline analysis assumes that the tax burden is given by a quadratic loss function for the tax rate. As shown by Equation (13), this implies that the terms in mean taxes and tax volatility are given equal coefficients in the calculation of the tax burden. This section investigates how important this assumption is by using a more general specification for the tax burden that allows the weights attached to these terms to vary.

In particular, consider a period loss function of the form $TB_t = (\tau_{c,t}/\tau_c)^b$, where $b \geq 1$ is a constant parameter. This general specification nests the baseline specification as a special case when $b = 2$, with deviations from this value changing the relative importance of the volatility effect. The expected tax burden is $E[TB] = E(\tau_{c,t}/\tau_c)^b$, which can be decomposed into mean and volatility effects through a second-order approximation around $\tau_{c,t} = E[\tau_{c,t}]$:

$$E[TB_t] \approx \underbrace{(E[\tau_{c,t}/\tau_c])^b}_{\text{Mean effect}} + \underbrace{\frac{b(b-1)(E[\tau_{c,t}])^{b-2}}{2(\tau_c)^b}\text{var}[\tau_{c,t}]}_{\text{Volatility effect}} \qquad (19)$$

For $b > 2$, this specification places a higher relative weight on the variance of taxes than the baseline specification and a lower weight on the mean of taxes; to see this, compare Equation (19) and Equation (13). Conversely, when $b < 2$ there is a lower relative weight on the variance of taxes and a higher weight on mean taxes. The special case of $b = 1$ implies that mean taxes matter for the tax burden but the variance of taxes does not. To investigate the robustness of the baseline conclusions, three different values were considered: $b = 2.3$, $b = 1.7$ and $b = 1.5$. The results are reported in Figure 5.

For all specifications it continues to be the case that issuing both indexed and nominal debt minimizes the expected tax burden under nominal GDP targeting, while inflation targeting still minimizes the expected tax burden when only indexed debt is issued. In Specification 1, where the value of b is increased to 2.3, the tax burden is now minimized at a lower indexation share of 57.6% under nominal GDP targeting, because the volatility effect (which is smaller at lower indexation shares under nominal GDP targeting) now becomes more important relative to the mean effect. For the specifications with b =1.7 and b = 1.5, the indexation share at which the tax burden is minimized rises to 84.8% and 98.2%, respectively, because mean taxes are given a higher weight under these specifications, and the volatility effect a lower relative weight.

Hence, the baseline result that the tax burden is minimized at an interior indexation share under nominal GDP targeting and a corner solution under inflation targeting is quite robust. It should be noted, however, that the burden-minimizing indexation share under nominal GDP targeting will eventually become 100% as the value of b is reduced towards 1,[37] because this reduces the volatility effect sufficiently that the mean effect dominates. By comparison, the result that nominal GDP targeting lowers the tax burden relative to inflation targeting (except at very high indexation shares) is not so robust.

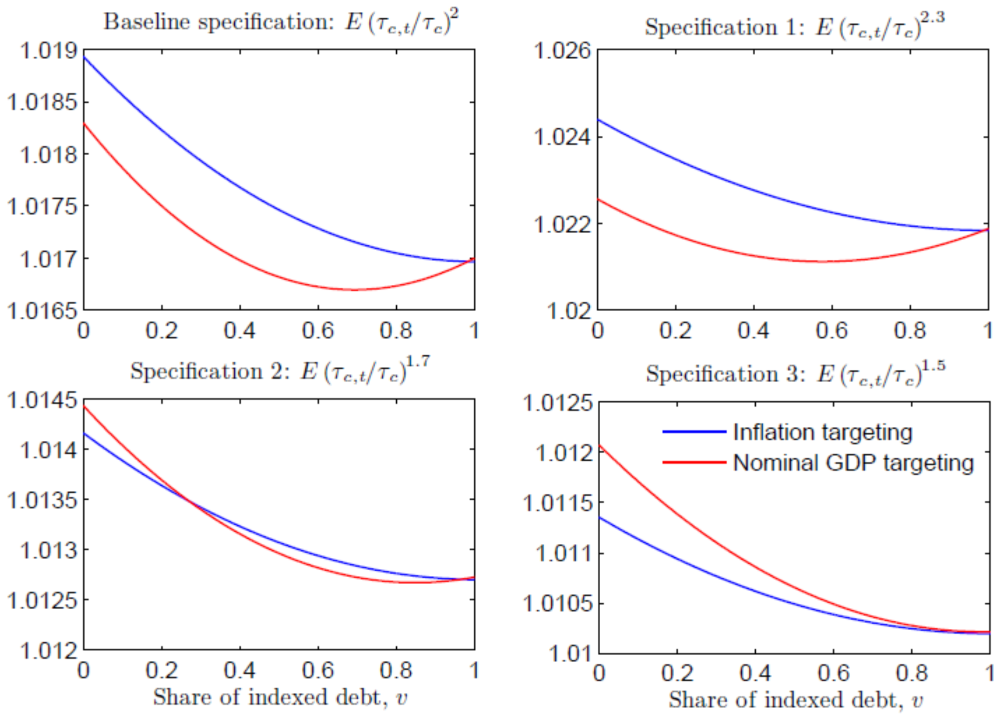

Fig 5 – Expected tax burden and indexation under alternative specifications of the tax burden.
Figure plots $E[TB]$ as the share of indexed debt, v, is varied from 0 to 1.

The specification with b = 2.3 preserves the baseline result because the variance of taxes now receives a higher weight than the mean, so that the tax burden under nominal GDP targeting is lowered further relative to that under inflation targeting (see first row, Figure 5). However, the results for other two specifications are more interesting. Under Specification 2, the value of b is reduced to 1.7. Even this relatively small reduction is sufficient to undo the result that the expected tax burden is lower under nominal GDP targeting except at high indexation shares (see second row, Figure 5).[38] In particular, the expected tax burden starts out being lower under inflation targeting, and this continues until an indexation share of around 25%. Once the indexation share is increased above this value the baseline result is restored: the

[37] Under the baseline calibration, the critical value at which this occurs is b = 1.477.

[38] Under the baseline calibration, the expected tax burden is lowered under nominal GDP targeting (except at high indexation shares) provided that $b \geq 1.81$.

expected tax burden becomes lower under nominal targeting until very high indexation shares are reached. Thus, nominal GDP targeting will only lower the expected tax burden relative to inflation targeting for quite a limited range of indexation shares in this case.[39]

With b = 1.5 we see a more dramatic change: the expected tax burden is lower under inflation targeting than nominal GDP targeting for all indexation shares, albeit that the difference is very small at indexation shares close to 100%. This happens because there is a sufficiently high weight on mean taxes that, under nominal GDP targeting, the higher level of taxes under nominal GDP targeting is no compensated for by the reduction in tax volatility. This result shows that the impact of nominal GDP targeting on the tax burden will depend crucially on the assumed loss function for the tax burden. In particular, nominal GDP targeting will lower the expected tax burden relative to inflation targeting (except at high indexation shares) if the loss function is quadratic or if it places a similar, or higher, relative weight on tax volatility to the quadratic case. Otherwise, the ranking between inflation targeting and nominal GDP targeting will be ambiguous – or, if the weight on tax volatility is low enough, inflation targeting will lower the expected tax burden relative to nominal GDP targeting.

5.3 Sensitivity analysis

5.3.1. Parameter sensitivity analysis

To investigate robustness, each of the model parameters was assigned a 'high' and 'low' value, as reported in the first column of Table 3. Results were then recomputed, giving a total of 22 sets of results for the alternative calibrations. For most cases, the main conclusions are remain intact. For instance, as under the baseline calibration, the expected tax burden is minimized by issuing only indexed debt under inflation targeting, a result which holds for all 22 cases. For nominal GDP targeting, there is considerable variation in terms of in the exact indexation share at which the expected tax burden is minimized, but this always occurs at an interior indexation share (see Table 3, Column 3), as in the baseline analysis.

In the large majority of the cases, the expected tax burden remains lower under nominal GDP targeting than inflation targeting up until very high indexation shares are reached (see Table 3, final column). However, in 2 of the 22 cases – high risk aversion, high productivity persistence[40] – there is no clear ranking of the expected tax burden under inflation targeting and nominal GDP targeting. Since these results look rather different to the baseline results, the relationship between the expected tax burden and the indexation share in each case is shown in Figure 6. In both cases the expected tax burden starts out lower under inflation targeting, in contrast to the baseline case. The expected tax burden then becomes lower under nominal GDP targeting once intermediate indexation shares of less than 30% are reached, remaining lower up until indexation shares close to 100%. Thus, although the tax burden

[39] A similar result was confirmed for the case of money in the utility function as a further robustness check.
[40] As shown in Table 3, the high risk aversion case sets γ = 2.5 and the high persistence case sets ρ_A = 0.70. Although these not a big changes relative to the baseline values (2 and 0.5 respectively), they are sufficient to cause a significant change in results as discussed below.

remains lower under nominal GDP targeting for a majority of indexation shares, the range of indexation shares for which this happens is relatively limited compared to the baseline case.

Table 3 – Parameter sensitivity analysis results

Parameter	**Value {Low, High}**	**Indexation share where $E(TB)^{NGDP}$ minimized {Low, High}**	**Indexation share where $E(TB)^{NGDP} > E(TB)^{IT}$ {Low, High}**
Capital share in output, α	0.27, 0.33	70.0%, 68.8%	99.2%, 96.6%
Private discount factor, β	0.45, 0.85	68.0%, 70.0 %	96.6%, 99.0%
Relative risk aversion, γ	1.5, 2.5	44.0%, 86.4%	99.8%, Multiple
Inverse Frisch elasticity, η	2.5, 3.5	70.6%, 68.4%	98.6%, 98.4%
Cash constraint parameter, θ	0.075, 0.125	70.2%, 68.6%	99.0%, 97.8%
Trend inflation, Π^*	1.62, 2.00	69.4%, 69.4%	98.2%, 98.6%
World real interest rate, r^*	1.6, 2.3	59.6%, 74.6%	97.2%, 99.2%
Government expenditure, g^*	0.11, 0.17	68.6%, 70.0%	98.4%, 98.6%
Productivity persistence, ρ_A	0.30, 0.70	54.2%, 83.2%	97.6%, Multiple
Std.(prod. innov.), σ_e	0.04, 0.06	77.2%, 62.4%	99.2%, 98.0%
Std.(money innov.), σ_ε	0.04, 0.06	60.8%, 75.8%	98.0%, 99.0%
Baseline calibration	**See Table 1**	**69.4%**	**98.4%**

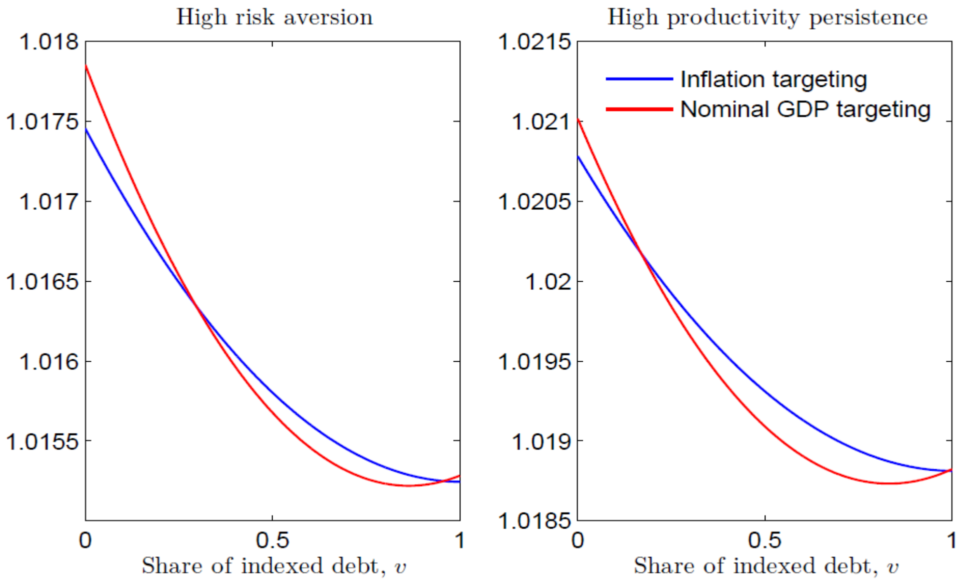

Fig 6 – Expected tax burden: high risk aversion and high productivity persistence calibrations. Figure plots $E[TB]$ in each case as the share of indexed debt, v, is varied from 0 to 1.

High risk aversion matters because this increases the magnitude of the mean effect. Specifically, high risk aversion magnifies the difference between the inflation risk premium under nominal GDP targeting and that under inflation targeting. As a result, mean taxes under nominal GDP targeting rise relative to mean taxes under inflation targeting. For this reason, the indexation share that minimizes the expected tax burden increases substantially to 86.4%.

High productivity persistence also matters because it makes the mean effect under nominal GDP targeting relatively more important. It does so because it implies larger movements in labour supply, which affect capital income received by the old via the marginal product of capital. This in turn raises inflation risk premium under nominal GDP targeting further, which implies an increase in mean taxes with no corresponding reduction in volatility. Hence, the expected tax burden is minimized at the somewhat higher indexation share of 83.2%.

5.3.2 Other robustness tests

Some further robustness checks are also in order. Firstly, notice from Table 3 that when the coefficient of relative risk aversion is reduced to 1.5, there is a very large fall in indexation share at which the expected tax burden is minimized under nominal GDP targeting, from 69% to 44%. If risk aversion is reduced further to log utility, the expected tax burden would be minimized at an indexation share of only 2.4% and the expected tax burden is *always* lower under nominal GDP targeting than under inflation targeting. Thus, low levels of risk aversion favour nominal debt over indexed debt, and nominal GDP targeting over inflation targeting.[41] Secondly, the baseline model assumes that money supply shocks are white noise, but it was found that relaxing this assumption does not make any substantive difference to the results. Third, as noted in Section 4.2, setting $\theta = 0$ so that money is absent makes the expected tax burden lower under nominal GDP targeting for all indexation shares below 100%, with the two regimes giving identical results when only indexed debt is issued. Of course, the same results hold $\theta_m = 0$ under money in the utility function (see Section 5.1).

Finally, if productivity shocks are absent, the expected tax burden will be minimized by issuing only indexed debt under *both* inflation targeting *and* nominal GDP targeting, and the expected tax burden is higher under nominal GDP targeting for *all* indexation shares. This shows that productivity shocks are crucial in driving the difference in results under inflation and nominal GDP targeting. Specifically, it is the presence of productivity shocks that allows nominal GDP targeting to reduce tax volatility by making the price level countercyclical, so that smaller movements in taxes are necessary to finance government expenditure than under inflation targeting where there is no stabilization of taxes in response to productivity shocks.

5.4 Nominal GDP growth targeting

The baseline analysis considers nominal GDP *level* targeting. If we consider nominal GDP *growth* targeting instead, the results are very similar (see Supplementary Appendix D). In particular, the expected tax burden is minimized at an indexation share of 70.0%, as compared to 69.4% in the baseline case. Moreover, the expected tax burden is lower than under inflation targeting unless the indexation share is 97.0% or higher, which compares to 98.4% in the baseline case. The reason for similar results is that the key difference in the two regimes is the response to past deviations from target under a level-targeting regime. Since

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[41] Similarly, a sufficiently high coefficient of relative risk aversion will favour indexed debt over nominal debt, and inflation targeting over nominal GDP targeting.

such deviations are part of *expected inflation*, unanticipated changes in inflation are very similar under the two regimes, and there is thus little difference between them in this setting.

6 Conclusion

This paper has investigated the impact of nominal GDP targeting on the tax burden using an overlapping generations model where monetary policy matters for distortionary taxes. The main mechanism in the model is that unanticipated inflation has real wealth effects on households who hold nominal government debt. The model was used to assess the tax burden under nominal GDP targeting and inflation targeting. It was found that nominal GDP targeting makes taxes less volatile than inflation targeting but raises average taxes. With a quadratic loss function, the expected tax burden is minimized under inflation targeting by issuing only indexed government debt. Nominal GDP targeting lowers the average tax burden relative to inflation targeting as it lowers the variance of taxes, and minimization of the tax burden requires indexed and nominal debt, in contrast to the case of inflation targeting.

These conclusions are quite robust within the overlapping generations model studied here. However, sensitivity analysis identified three factors which are crucial for the baseline results: risk aversion, productivity persistence and the loss function for the tax burden. Both high risk aversion and high productivity persistence can overturn the baseline result by making the expected tax burden lower under inflation targeting at relatively low indexation shares. On the other hand, the form of the loss function matters because, relative to inflation targeting, nominal GDP targeting move mean taxes and tax volatility in opposite directions. Overall, the results suggest that inflation targeting and nominal GDP targeting could have quite different implications for the tax burden, and that the impact will depend crucially on the share of indexed government debt.

Acknowledgements
The author would like to thank the Editor and an anonymous referee for useful comments that substantially improved the paper.

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Appendix

The inflation risk premium and monetary policy

This Appendix shows that the difference in expected real returns on nominal and indexed bonds can be interpreted as an inflation risk premium, and discusses how this premium relates to monetary policy and the share of indexed debt.

Letting $\widetilde{sdf}_{t+1} \equiv sdf_{t+1} \dfrac{(1+\tau_{c,t})}{(1+\tau_{c,t+1})}$, the first-order conditions for bonds, (6) and (7), imply that

$$1 = E_t[\widetilde{sdf}_{t+1} r_{t+1,n}] + \theta\tilde{\mu}_t \tag{A1}$$

$$1 = r_{t,f} E_t[\widetilde{sdf}_{t+1}] + \theta\tilde{\mu}_t \tag{A2}$$

where $\tilde{\mu}_t \equiv \dfrac{\mu_t(1+\tau_{c,t})}{U_c(c_{1,t})}$ and $sdf_{t+1} = \beta U_c(c_{2,t+1})/U_c(c_{1,t})$ is the stochastic discount factor.

Therefore,

$$irp = E_t[r_{t+1,n}] - r_{t,f} = -\frac{\mathrm{cov}_t[\widetilde{sdf}_{t+1}, r_{t+1,n}]}{E_t[\widetilde{sdf}_{t+1}]} = -\frac{\mathrm{cov}_t[\widetilde{sdf}_{t+1}, R_t/\Pi_{t+1}]}{E_t[\widetilde{sdf}_{t+1}]} \tag{A3}$$

The expression in Equation (A3) can be interpreted as an inflation risk premium because, in general, it is non-zero unless inflation risk is equal to zero. The numerical analysis in the paper focuses on the unconditional expectation of Equation (A3).

Using the same steps as above, but taking the unconditional expectation of (A1) and (A2) and dropping time subscripts for ease of interpretation, we have:

$$irp = E[r_n] - E[r_f] = -\frac{\mathrm{cov}[\widetilde{sdf}, r_n]}{E[\widetilde{sdf}]} = -\mathrm{cv}[\widetilde{sdf}] \times \mathrm{corr}[\widetilde{sdf}, r_n]\mathrm{std}[r_n] \tag{A4}$$

where *cv*[*x*] denotes the coefficient of variation of *x*, and std[*x*] is the standard deviation of *x*.

Monetary policy directly affects the correlation and standard deviation terms in (A4) through the real return on nominal debt, $r_n = R/\Pi$. The correlation term is negative under calibrations in this paper whenever some nominal debt is held, since an unanticipated inflation lowers the consumption of the old and the taxes that they face, raising marginal utility. The correlation falls in absolute value as the share of indexed debt rises because household portfolios (and the taxes they face) are less vulnerable to inflation risk as more inflation-indexed debt is held.

Under inflation targeting, the correlation is equal to *zero* when only indexed debt is held because this ensures that no part of the real wealth of the old is influenced by unanticipated inflation. By contrast, the correlation remains negative under nominal GDP targeting, so the inflation risk premium is positive even if only indexed debt is held (see Figure 3, bottom row). The reason is that monetary policy responds to negative productivity shocks with unanticipated inflation, so that inflation negatively covaries with the return on capital. This covariance between inflation and marginal utility is present even if no nominal debt is held.

Supplementary Appendix (For online publication only)

Section A – The binding legal constraint on money holdings

It is shown in this section that the constraint on real money holdings binds with strict equality if the gross money return on a nominal bonds exceeds 1.

Proposition: The constraint binds with strict equality when $R_t > 1$

Proof.

By equations (6) and (8), the Lagrange multiplier on the cash constraint is given by

$$\mu_t = \beta E_t\left[\frac{U_c(c_{2,t+1})(r_{t+1,n} - r_{t+1,m})}{(1+\tau_{c,t+1})}\right] \tag{A1}$$

Since the real return on nominal bonds is $r_{t+1,n} = R_t / \Pi_{t+1} = R_t r_{t+1,m}$, we can also write

$$\mu_t = \beta E_t\left[\frac{U_c(c_{2,t+1})(R_t - 1)r_{t+1,m}}{(1+\tau_{c,t+1})}\right] = \beta(R_t - 1)E_t\left[\frac{U_c(c_{2,t+1})}{(1+\tau_{c,t+1})\Pi_{t+1}}\right] \tag{A2}$$

since the nominal yield on nominal government bonds, R_t, is known at the end of period t.

The Kuhn-Tucker conditions associated with μ_t are as follows:

$$\left\{ \mu_t \geq 0 \quad \text{and} \quad \mu_t(m_t - \theta(1+\tau_{c,t})c_{1,t}) = 0 \right\} \tag{A3}$$

The second condition in (A3) is the complementary slackness condition. It implies that the cash constraint will bind iff $\mu_t > 0$ for all t. By (A2), this holds if $R_t > 1$for all t. **Q.E.D.**

Section B – Impulse responses under the baseline calibration

B1 – Impulse responses to a money supply innovation under inflation targeting (IT) and nominal GDP targeting (NGDP)

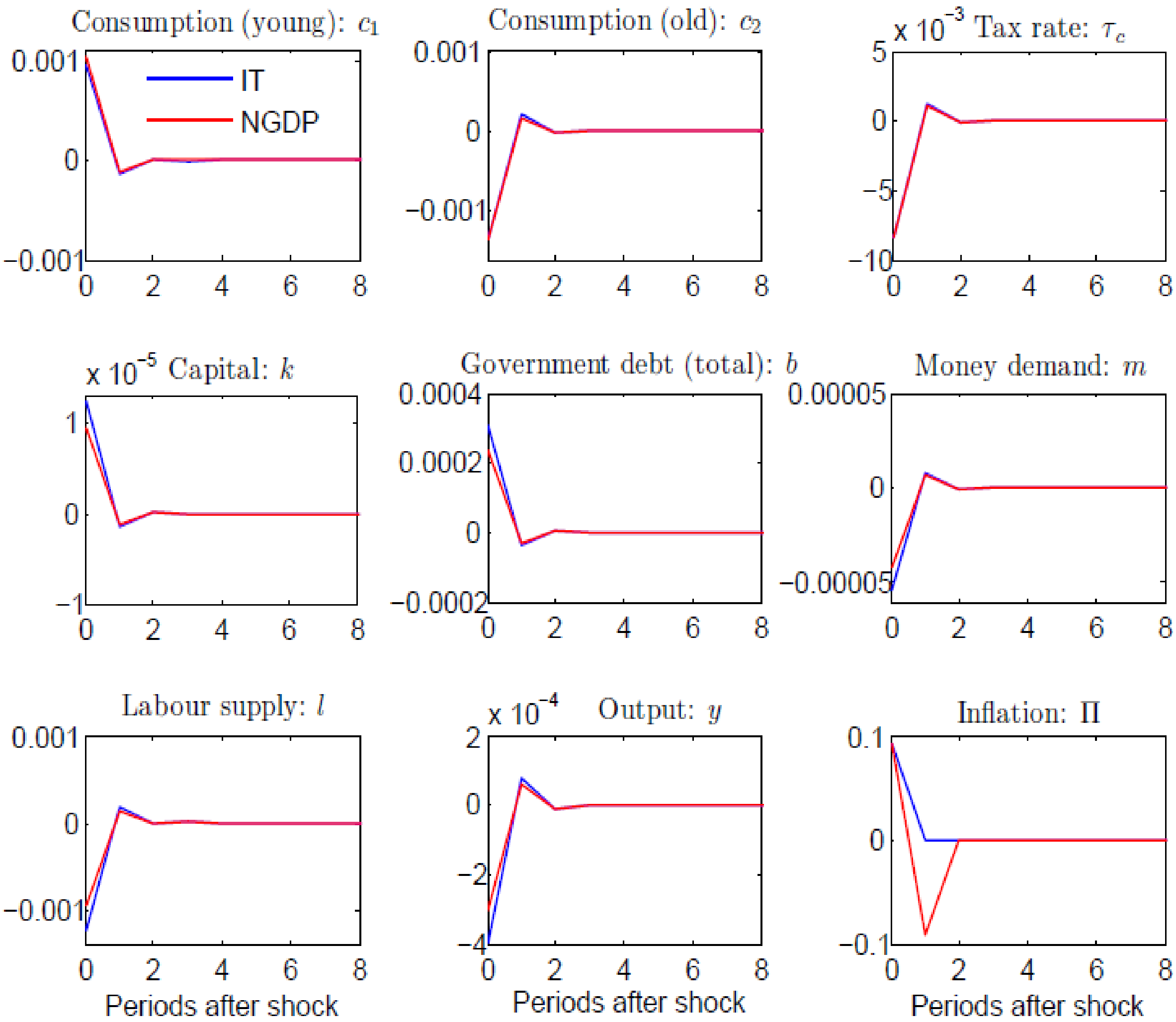

Figure B1 – Impulse responses to a monetary innovation under inflation and nominal GDP targeting. NB. When only one line is visible this indicates that the IRFs are essentially identical under both regimes.

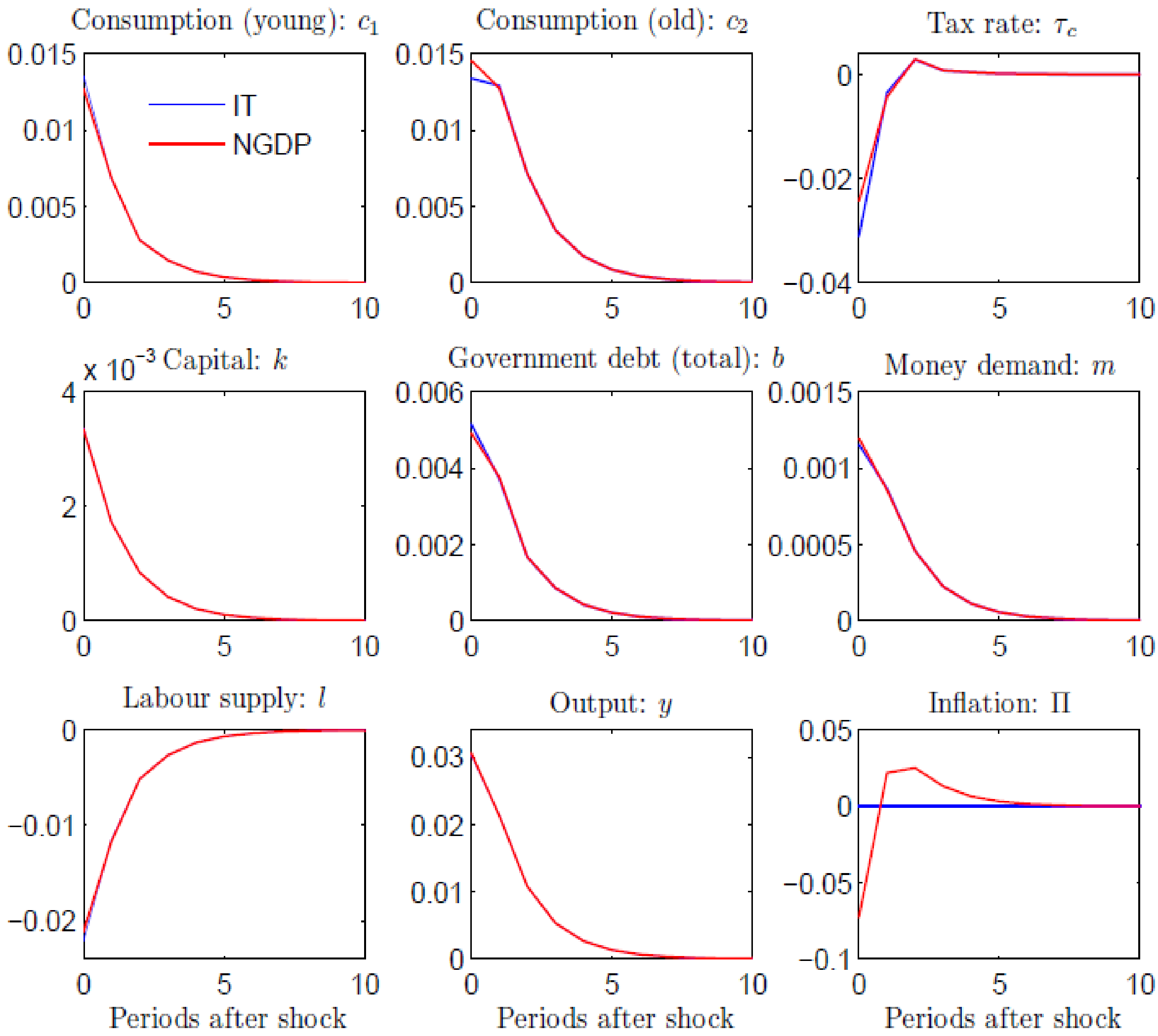

Figure B2 – Impulse responses to a productivity innovation under inflation and nominal GDP targeting. NB. When only one line is visible this indicates that the IRFs are essentially identical under both regimes.

Section C – Results under money in the utility function

The relative weight on money holdings, θ_m, is set at 0.01 so that real money balances are 4% of GDP as in the baseline model. The utility curvature parameter γ_m is set equal to 1.5.

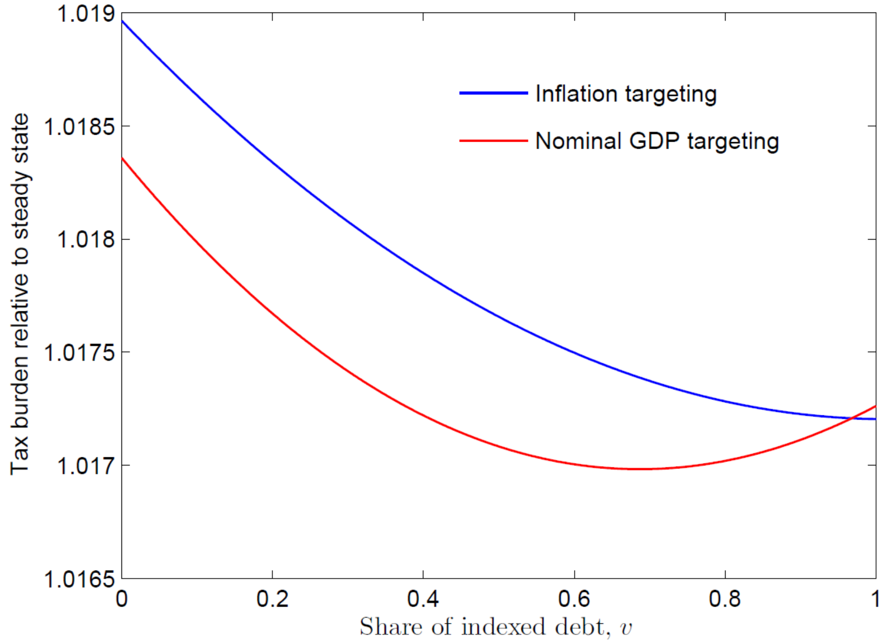

Fig C1 – Expected tax burden and indexation (MIUF). Figure plots $E[TB]$ as the share of indexed debt, v, is varied from 0 to 1. The steady state value is identical under both regimes and does not vary with the share of indexed debt. MIUF denotes 'money in the utility function'.

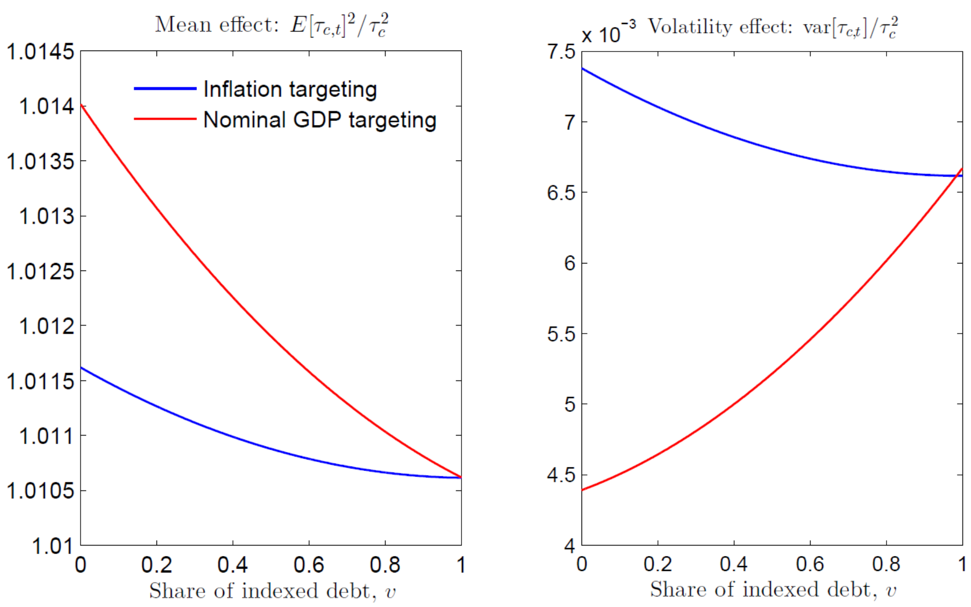

Fig C2 – Decomposition of expected tax burden into mean and volatility effect (MIUF). Figure plots the mean effect and the volatility effect as the share of indexed debt is varied from 0 to 1. The sum of the mean and volatility effects equals expected tax burden plotted in Fig C1. MIUF denotes 'money in the utility function'.

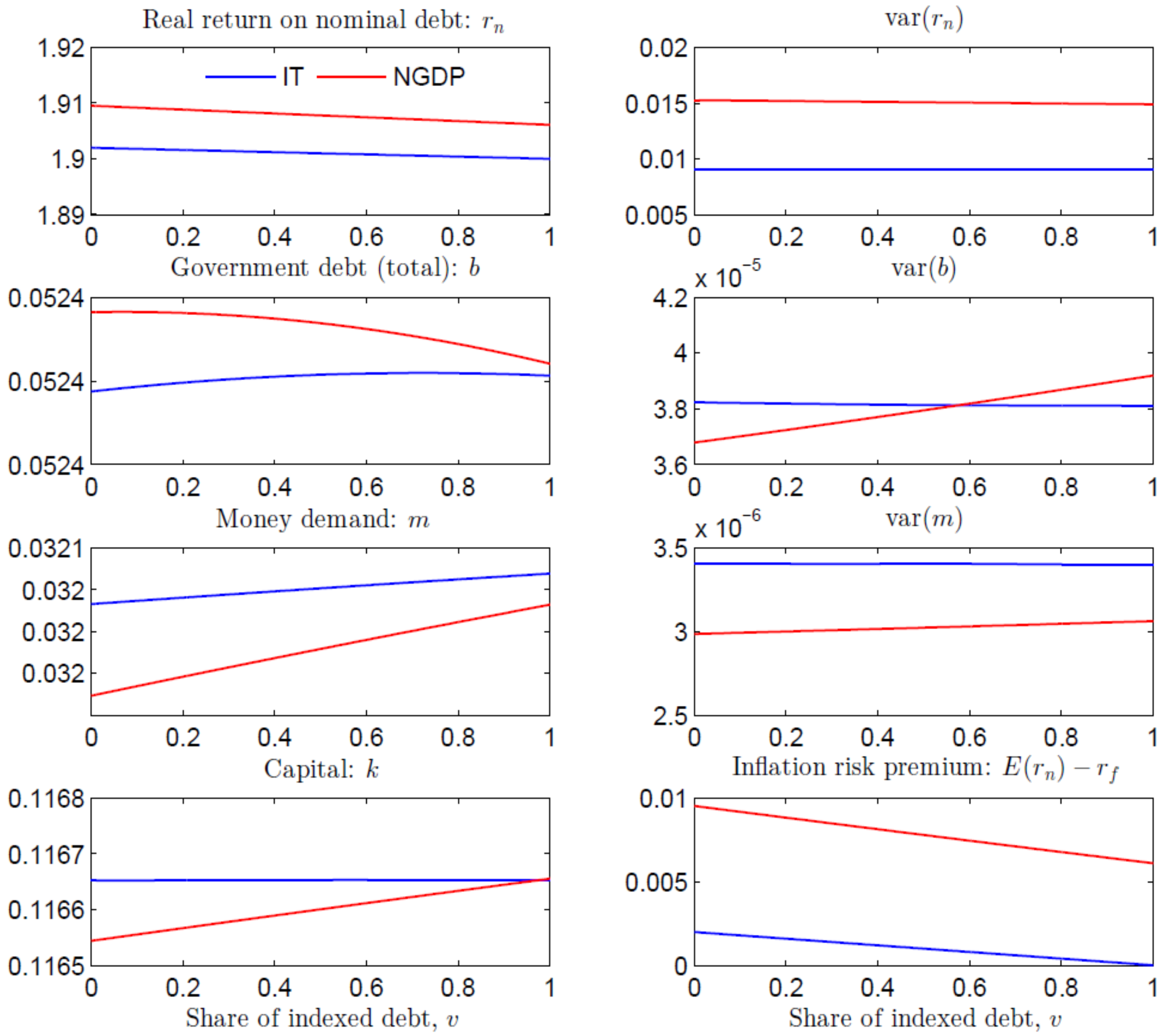

Fig C3 – Real variables and indexation (MIUF). Figure plots the unconditional moments of variables under inflation targeting and nominal GDP targeting as the share of indexed debt is varied from 0 to 1. MIUF denotes ‘money in the utility function’.

Section D – Nominal GDP *growth* targeting vs nominal GDP level targeting

Since 'bygones are bygones' under a nominal GDP growth targeting regime, the nominal money supply evolves according to the following rule:

$$M_t = M_{t-1}\Pi_t^*(m_t / m_{t-1})\exp(\varepsilon_t) \quad \text{(D1)}$$

As under level targeting, the desired inflation rate is $\Pi^* y_{t-1}/ y_t$, . Hence, inflation under nominal GDP growth targeting is given by

$$\Pi_t^{NGDP,GR} = \Pi^*[y_{t-1} / y_t]\exp(\varepsilon_t) \quad \text{(D2)}$$

The figures below present results for this case under the baseline calibration (see Section 4.1).

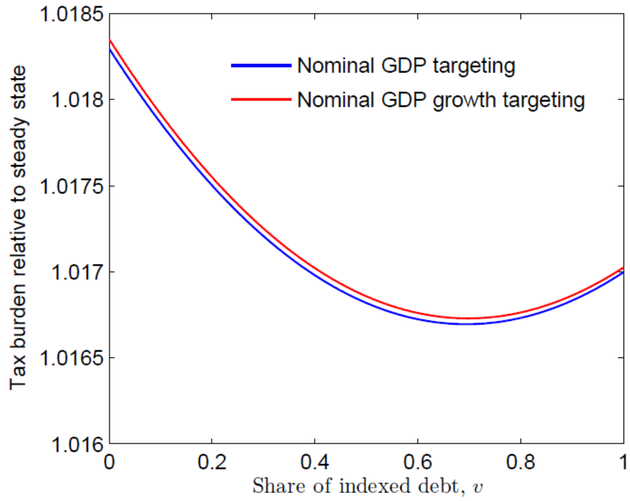

Fig D1 – Expected tax burden and indexation. Figure plots $E[TB]$ as the share of indexed debt, v, is varied from 0 to 1. The steady state is identical under both regimes and does not vary with the share of indexed debt.

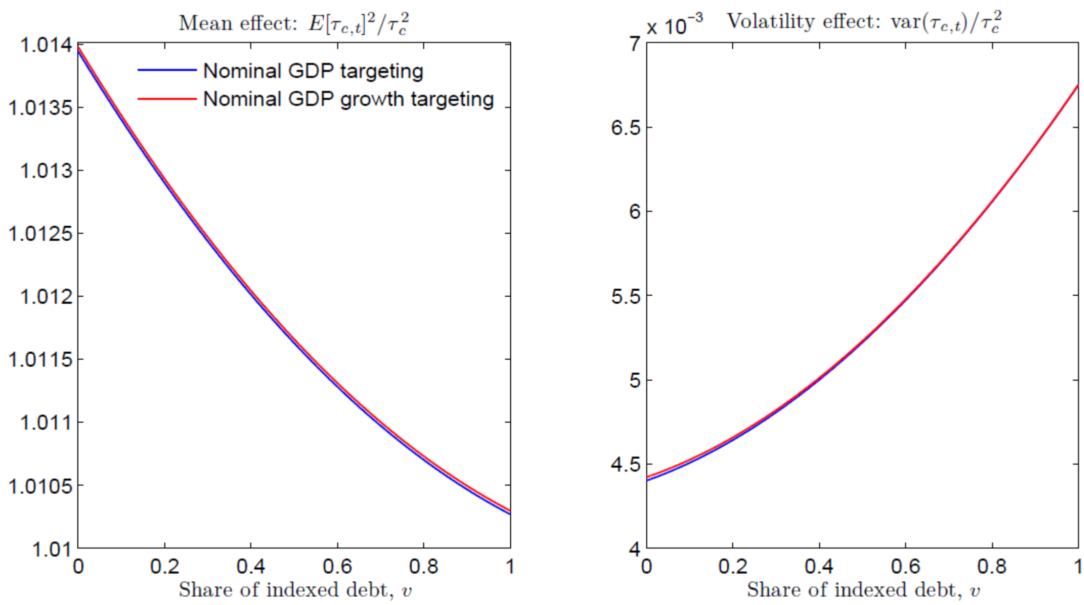

Fig D2 – Decomposition of expected tax burden into mean and volatility effect. Figure plots the mean effect and the volatility effect as the share of indexed debt is varied from 0 to 1.

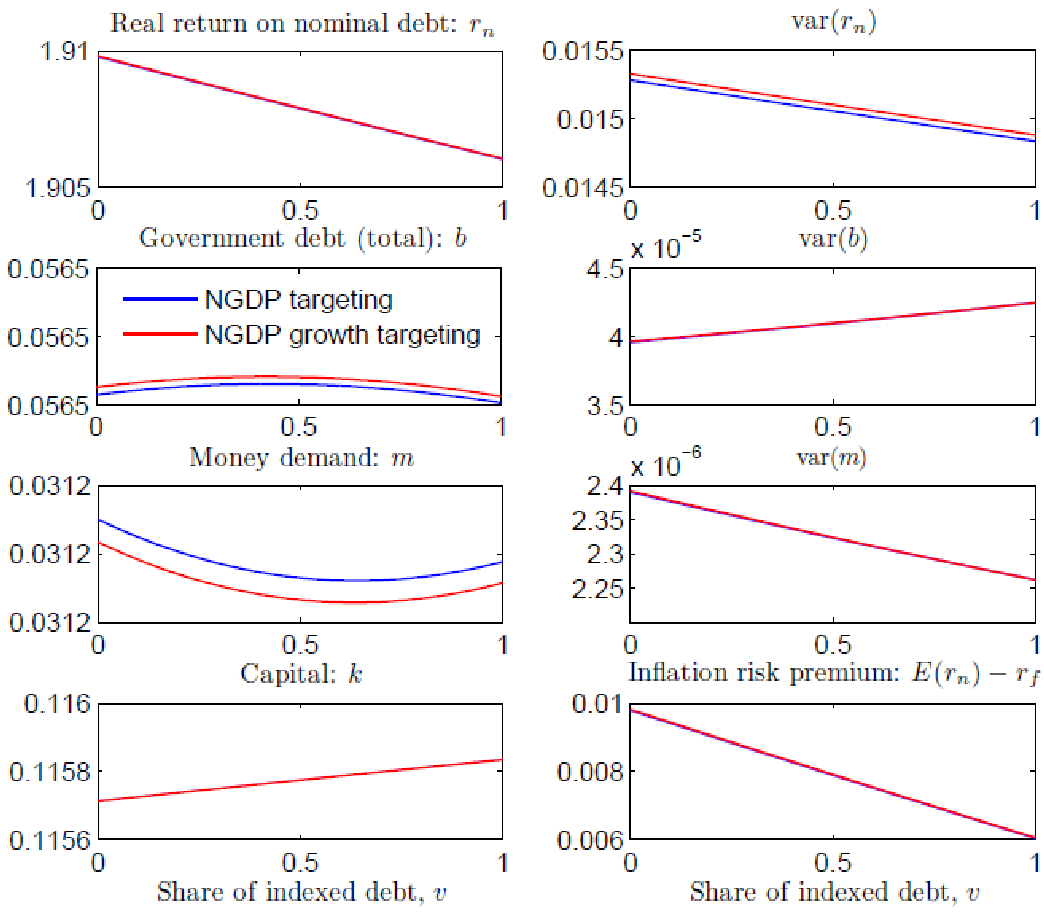

Fig D3 – Real variables and indexation. Figure plots the unconditional moments of variables under nominal GDP targeting and nominal GDP growth targeting as the share of indexed debt is varied from 0 to 1.